Test 1 Review

Note Title 2/8/2012

We consider Exam 1a

Note: The actual exam will be LOTS shorter!

Find the plane problems:

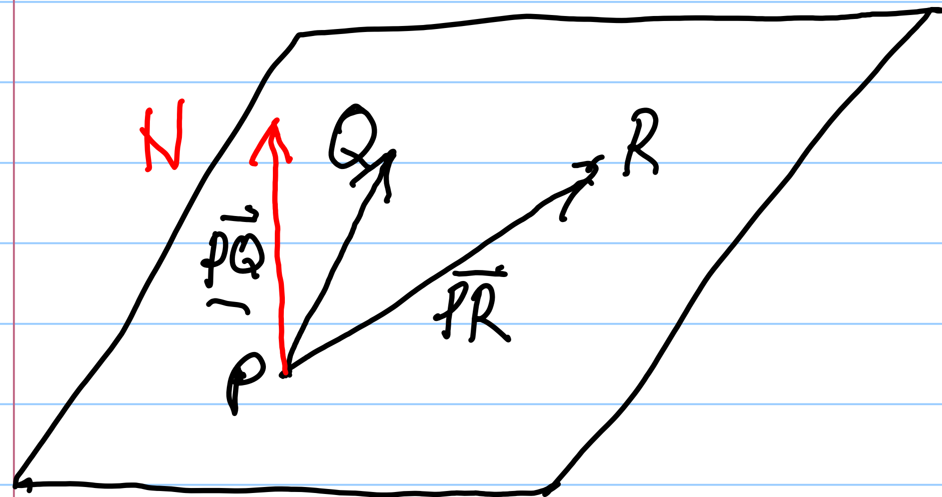
#5, #12, #16, #17

All such problems can be reduced to the technique of #12

Find the plane containing $P = (1,2,1)$, $Q = (2,-1,0)$

$R = (3,3,1)$

Step 1: Find N



$$Q - P = (2,-1,0) - (1,2,1)$$
$$= (1,-3,-1)$$

$$\overrightarrow{PQ} = i - 3j - k$$

$$R - P = (3,3,1) - (1,2,1)$$
$$= 2i + j + 0k$$

$$\vec{N} = \overrightarrow{PQ} \times \overrightarrow{PR}$$

$$= \begin{vmatrix} i & j & k \\ 1 & -3 & -1 \\ 2 & 1 & 0 \end{vmatrix}$$

$$= i - 2j + 7k = \vec{N}$$

Plane:

$x - 2y + 7z = d$

To find d, plug in any point on plane e.g. $(1,2,1)$

$d = 1 - 2\cdot 2 + 7\cdot 1 = 4$

$$\boxed{x - 2y + 7z = 4}$$

Most other "find the plane" problems can be solved as above by finding 3 non-colinear points on the plane e.g.

#5 Find the plane containing both the point $(1,2,3)$ and the line $x=3t,\ y=1+t,\ z=2-t$

Solution:

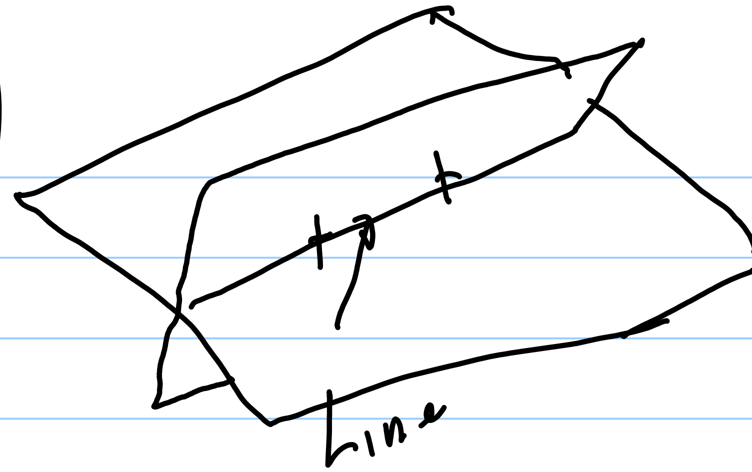
$t=0 \Rightarrow (0,1,2)$ on plane

$t=1 \Rightarrow (3,2,1)$ on plane

proceed as above.

#16 Find the plane that contains $(-1,2,1)$ and the line of intersection of the planes $x+y-z=2$ and $2x-y+3z=1$

$(-1,2,1)$



Solution:

Line: $\quad x+y-z=2$

$\qquad\quad 2x-y+3z=1$

Find two points on the line of intersection. To do this pick two "random" values of z & solve for x & y. Eg.

$z=0$:

$$\begin{aligned} x+y &= 2 \\ 2x-y &= 1 \end{aligned} \implies x=1,\ y=1$$

$$Q = (1, 1, 0)$$

$z=1$
$$\begin{aligned} x+y-1 &= 2 \\ 2x-y+3 &= 1 \end{aligned}$$

$$\begin{aligned} x+y &= 3 \\ 2x-y &= -2 \end{aligned} \implies \begin{aligned} x &= \tfrac{1}{3} \\ y &= \tfrac{8}{3} \end{aligned}$$

$$R = \left(\tfrac{1}{3}, \tfrac{8}{3}, 1\right)$$

#17. Plane determined by two given lines:

Solution:

Simple. Just find 2 points on the first line and one on the second & proceed as before

Curvature problems:

Most curvature problems can be solved with the formula

$$\kappa = \frac{|\vec{v} \times \vec{a}|}{|\vec{v}|^3}$$

$\vec{v}$ = velocity
$\vec{a}$ = acceleration

This formula will be provided on the last page of the exam.

Often you will first need to parametrize the curve to find $\vec{v}$ and $\vec{a}$.

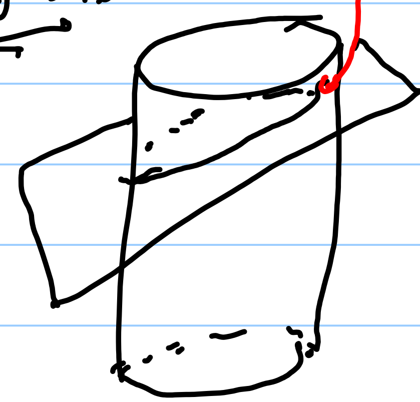
Note! I probably would allow you to just find $\vec{v}$ + $\vec{a}$ and then say "plug the result into the formula!" What I want will be made clear on test

14. Let C be the intersection of $x^2+y^2=16$ and $x+y+z=5$. Find the curvature of C at $(0,4,1)$

$x=4\cos t$ $y=4\sin t$

C

$x^2+y^2=16$



Solution:

Parametrize C

Let $x=4\cos t$, $y=(\sin t)4$

On C , $z=5-x-y$

$=5-4\cos t-4\sin t$

$\langle x,y,z\rangle=\langle 4\cos t, 4\sin t, 5-4\cos t-4\sin t\rangle$

$=\vec{r}(t)$

4

$\vec{r}\,' = \vec{v} = \langle -\sin t, \cos t, \sin t - \cos t \rangle$

etc.

$\vec{r}\,'' = \vec{a} = \langle -\cos t, -\sin t, \cos t + \sin t \rangle$

@ $t = \cancel{0}$, $\vec{v} = \langle 0, 1, -1 \rangle$

π $\quad \vec{a} = \langle -1, 0, 1 \rangle$

Plug into curvature formula.

Graphing in 3-d.

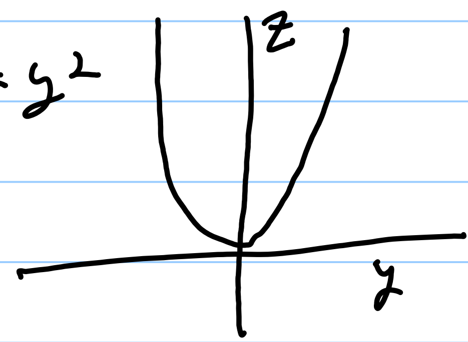
You will not need to know the names of the graphs. But you will need to be able to do problems such as 1–12 on P. 748 where you match the formula with the graph.

Problem #3 Which picture most closely matches the graph of $\boxed{y^2 - x^2 = z}$ (pictures will be provided).

Solution!

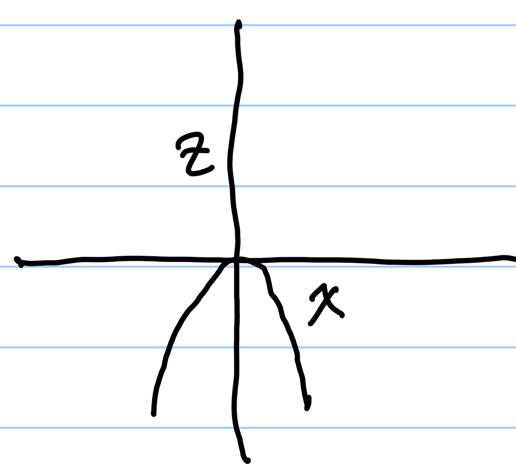
x trace $\quad z = y^2$

$x = 0$

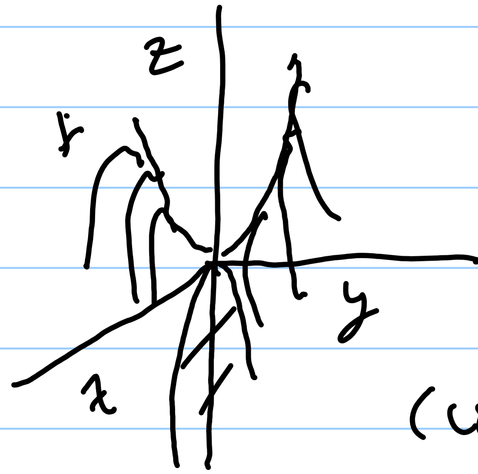


y trace

$y = 0 \quad z = -x^2$



Goes up on y axis, down on x-axis



saddle

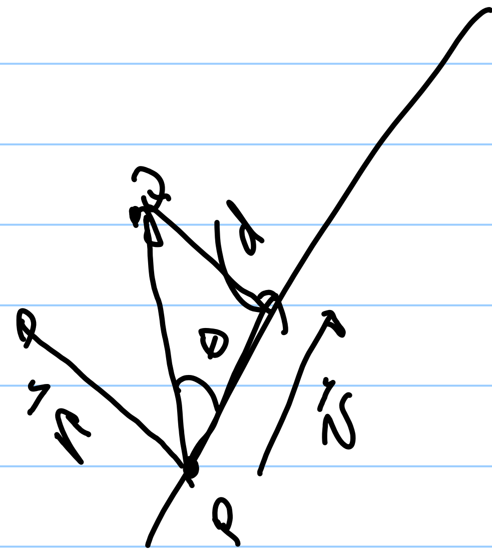
Non of the pictures work.

(compare traces)

I will not ask you to do a problem such as #15.

I will not ask about projectile motion (#18)

What Else?

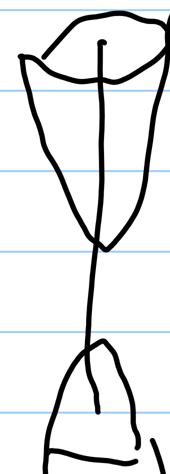


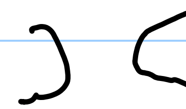
ans $d = |\vec{PQ}| \sin\theta$ ~~$\cos\theta$~~

$= \cancel{|\vec{PQ}|} \dfrac{|\vec{PQ} \times \vec{v}|}{\cancel{|\vec{PQ}|}\,|\vec{v}|}$

$z^2 - x^2 + y^2 = 1$

$z^2 - x^2 - y^2 = 1$





$z^2 - x^2 = 1$